Energy Distribution of a Charged Regular Black Hole

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Abstract

We calculate the energy distribution of a charged regular black hole by using the energy-momentum complexes of Einstein and Møller.

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1 INTRODUCTION

One of the most interesting and intricate problem of relativity is the energy-momentum localization. The different attempts at constructing an energy-momentum density don't lead to a generally accepted expression. However, there are various energy-momentum complexes including those of Einstein [1], Tolman [2], Landau and Lifshitz [3], Papapetrou [4], Bergmann [5], Weinberg [6] and Møller [7]. Cooperstock [8] gave his opinion that the energy and momentum are confined to the regions of non-vanishing energy-momentum tensor of the matter and all non-gravitational fields. Although, the energy-momentum complexes are coordinate dependent they can give a reasonable result if calculations are carried out in Cartesian coordinates. Some interesting results obtained recently lead to the conclusion that different energy-momentum complexes give the same energy distribution for a given space-time [9]-[16].

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We calculate the energy distribution of a charged regular black hole by using the energy-momentum complexes of Einstein and Møller. We use the geometrized units ($G = 1, c = 1$) and follow the convention that Latin indices run from 0 to 3.

2 Energy in the Einstein prescription

The Reissner-Nordström (RN) metric is the only static and asymptotically flat solution of the Einstein-Maxwell equations and it represents an electrically charged black hole. The metric is given by

$$ds^2 = A(r)dt^2 - B(r)dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{1}$$

where

$$A(r) = B^{-1}(r) = 1 - \frac{2M}{r} + \frac{q^2}{r^2} \tag{2}$$

and q and M are the electric charge and, respectively, the mass of the black hole.

A solution to the coupled system of the Einstein field and equations of the nonlinear electrodynamics was recently given by E. Ayón-Beato and A.Garcia (ABG) [17]. This solution represents a regular black hole with mass M and electric charge q and avoids thus the singularity problem. Also, the metric asymptotically behaves as the Reissner-Nordström solution. The usual singularity of the RN solution, at $r = 0$, has been smoothed out and now it simply corresponds to the origin of the spherical coordinates. The line element is given by (1) with

$$A(r) = B^{-1}(r) = 1 - \frac{2M}{r}(1 - \tanh(\frac{q^2}{2Mr})). \tag{3}$$

If the electric charge vanishes we reach the Schwarzschild solution. At large distances (3) resembles to the Reissner-Nordström solution and can be written

$$A(r) = B^{-1}(r) = 1 - \frac{2M}{r} + \frac{q^2}{r^2} - \frac{q^6}{12M^2r^4} + O(\frac{1}{r^6}). \tag{4}$$

We obtain the energy distribution associated with the solution given by (1) and (3) in the Einstein and Møller prescriptions.

The Einstein energy-momentum complex [1] is given by

$$\Theta_i^{\ k} = \frac{1}{16\pi} H_{i\ \ ,l}^{\ kl}, \tag{5}$$

where

$$H_i^{\ kl} = -H_i^{\ lk} = \frac{g_{in}}{\sqrt{-g}}[-g[g^{kn}g^{lm} - g^{\ln}g^{km})]_{,l}. \tag{6}$$

$\Theta_0^{\ 0}$ and $\Theta_\alpha^{\ 0}$ are the energy and, respectively, the momentum components.

The Einstein energy-momentum complex satisfies the local conservation laws

$$\frac{\partial \Theta_i^{\ k}}{\partial x^k} = 0. \tag{7}$$

The energy and momentum in the Einstein prescription are given by

$$P_i = \iiint \Theta_i^{\ 0} dx^1 dx^2 dx^3. \tag{8}$$

Using the Gauss theorem we obtain

$$P_i = \frac{1}{8\pi} \iint H_i^{\ 0\alpha} n_\alpha dS, \tag{9}$$

where $n_\alpha = \left(\frac{x}{r}, \frac{y}{r}, \frac{z}{r}\right)$ are the components of a normal vector over an infinitesimal surface element $dS = r^2 \sin\theta d\theta d\varphi$.

The required nonvanishing components of $H_i^{\ kl}$ for the line element given by (1) and (3) are

$$\begin{aligned} H_0^{\ 01} &= -\tfrac{4Mx(-1+\tanh(\frac{q^2}{2Mr}))}{r^3}, \\ H_0^{\ 02} &= -\tfrac{4My(-1+\tanh(\frac{q^2}{2Mr}))}{r^3}, \\ H_0^{\ 03} &= -\tfrac{4Mz(-1+\tanh(\frac{q^2}{2Mr}))}{r^3}. \end{aligned} \tag{10}$$

Using the above expressions in (9) we obtain

$$E(r) = M(1 - \tanh(\frac{q^2}{2Mr})). \tag{11}$$

From (11) it results that if $q = 0$ we have the energy of a Schwarzschild black hole.

After some calculations, we get the energy distribution of the ABG black hole

$$E(r) = M - \frac{q^2}{2r} + \frac{q^6}{24M^2r^3} - \frac{q^{10}}{240M^4r^5} + O(\frac{1}{r^6}). \tag{12}$$

Also, (12) can be written

$$E(r) = E_{RN}(r) + \frac{q^6}{24M^2r^3} - \frac{q^{10}}{240M^4r^5} + O(\frac{1}{r^6}). \tag{13}$$

3 Energy in the Møller prescription

The Møller energy-momentum complex [7] no needs to carry out calculations in Cartesian coordinates so we can calculate in any coordinate system.

Møller's energy-momentum complex is given by

$$M_i{}^k = \frac{1}{8\pi} \cdot \frac{\partial \chi_i{}^{kl}}{\partial x^l}, \tag{14}$$

where

$$\chi_i{}^{kl} = \sqrt{-g} \left(\frac{\partial g_{in}}{\partial x^m} - \frac{\partial g_{im}}{\partial x^n} \right) g^{km} g^{ln}. \tag{15}$$

The energy in the Møller prescription has the expression

$$E = \iiint M_0{}^0 dx^1 dx^2 dx^3 = \frac{1}{8\pi} \iiint \frac{\partial \chi_0{}^{0l}}{\partial x^l} dx^1 dx^2 dx^3. \tag{16}$$

For the line-element given by (1) and (3) the $\chi_0{}^{01}$ component is given by

$$\chi_0{}^{01} = \frac{(2Mr - 2Mr\tanh(\frac{q^2}{2Mr}) - q^2 + q^2\tanh^2(\frac{q^2}{2Mr}))\sin\theta}{r}. \tag{17}$$

From (16) and (17) and applying the Gauss theorem we obtain the energy distribution

$$E(r) = M(1 - \tanh(\frac{q^2}{2Mr})) - \frac{q^2}{2r}(1 - \tanh^2(\frac{q^2}{2Mr})). \tag{18}$$

Also, from (18) we have

$$E(r) = M - \frac{q^2}{r} + \frac{q^6}{6M^2r^3} - \frac{q^{10}}{40M^4r^5} + O(\frac{1}{r^6}). \quad (19)$$

4 Discussion

Bondi [18] sustained that a nonlocalizable form of energy is not admissible in relativity so its location can in principle be found. We obtain the energy distribution of a charged regular black hole by using the energy-momentum complexes of Einstein and Møller. From (13) it results that in the Einstein prescription the first two terms in the expression of the energy correspond to the Penrose quasi-local mass definition (evaluated by Tod) [19] . The Møller formalism provides in the expression of the energy (19) a term $M - \frac{q^2}{r}$ which agrees with the Komar [20] prescription. Also, the Møller energy-momentum complex allows to make the calculations in any particular system of coordinates. However, the energy of the ABG black hole obtained by using the Einstein energy-momentum complex contains the quantity $M - \frac{q^2}{2r}$ which is agreeing with linear theory. Our result sustains the Virbhadra opinion [21] that "the Einstein energy-momentum complex is in a better bill of health" and gives acceptable results for many space-times. Also, Chang, Nester and Chen [22] showed that the energy-momentum complexes are actually quasilocal and legitimate expressions for the energy-momentum.

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